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Welcome to Module 3 - Long Term Actuarial Mathematics of the Certified Actuarial Analyst qualification

The Certified Actuarial Analyst (CAA) is a professional qualification from CAA Global.

It is designed to provide you with a technical skills qualification if you:

• work alongside actuaries – in areas such as data analysis, pricing and marketing

• work in the wider financial services area – perhaps you already have other qualifications, and would like to develop a skill set that will mark you out in a competitive environment

• work in a service centre environment – the analytical skills you’ll learn can then be added to your business knowledge

• have strong maths skills, and you want to and learn on the job rather than going to university.

The aim of the Module 3 Long Term Actuarial Mathematics exam is to provide grounding in the aspects of statistics that are of relevance to actuarial work and in stochastic processes and survival models.

This Resource guide for Module 3 gives you the syllabus you will cover for the exam, and details of some online and other resources that will help you study for the Module 3 exam. There is also a specimen exam paper giving examples of the type of questions you will be asked.
The Certified Actuarial Analyst qualification

There are seven exams which you will need to complete for the qualification:

<table>
<thead>
<tr>
<th>Module title</th>
<th>Assessed by</th>
</tr>
</thead>
<tbody>
<tr>
<td>Fundamental Mathematics &amp; Statistics – Module 0</td>
<td>2 hour Computer Based Assessment</td>
</tr>
<tr>
<td>Finance and Financial Mathematics – Module 1</td>
<td>2 hour Computer Based Assessment</td>
</tr>
<tr>
<td>Statistics and Models – Module 2</td>
<td>2 hour Computer Based Assessment</td>
</tr>
<tr>
<td><strong>Long Term Actuarial Mathematics – Module 3</strong></td>
<td>2 hour Computer Based Assessment of 60 questions</td>
</tr>
<tr>
<td>Short Term Actuarial Mathematics – Module 4</td>
<td>2 hour Computer Based Assessment</td>
</tr>
<tr>
<td>Models and Audit Trails – Module 5</td>
<td>3 hour examination</td>
</tr>
<tr>
<td>Online Professional Awareness Test (PAT)</td>
<td>90 minute examination</td>
</tr>
</tbody>
</table>

In addition to passing the above exams, you must complete at least one year of relevant work experience (Work-based skills).
The syllabus for the Module 3 exam

The mathematical topics covered by the Module 3 exam are:

- Simple Assurance and Annuity contracts; formulae for the mean and variance of the present value of payments under these contracts (assuming deterministic interest)
- Practical methods of evaluating the expected values of contracts as defined in Topic 1 of the syllabus.
- Gross premiums for Life insurance and annuity contracts
- Gross Premium Reserves
- Calculating gross premiums and gross premium reserves for increasing and decreasing benefits and annuities using ultimate or select mortality
- Straightforward functions involving two lives
- Modelling cash flows contingent on risks
- Techniques of discounting emerging costs for use in pricing, reserving, and assessing profitability
- The principal forms of heterogeneity within a population and the ways in which selection can occur.

You can find the full Module 3 syllabus in Appendix 1 of this Resource guide.

Studying for the Module 3 exam

Recommended study hours

We recommend that you spend 125-150 hours studying to prepare for the exam.
Tuition

BPP ACTUARIAL EDUCATION

BPP Actuarial Education (ActEd),
provides online study material for this exam.
Details of their training materials and services are available on their website.
Website: www.bppacted.com
Email: ActEd@bpp.com
Tel: +44 (0)1235 550 005

Please note
Education providers are listed here for information purposes. CAA Global has not
assessed the quality of the services provided.

Assessment of the Module 3 exam

The Module 3 exam is assessed by a 2 hour computer
based exam containing 60 multiple choice questions.

Before you start your 2 hour exam you will have an
extra 15 minutes for:

• reading the exam instructions, and
• working through some basic sample questions so
  that you become familiar with the format of the exam.

You will also need to sign a statement of confidentiality
in relation to the exam materials.

Pass standards for the exam are set by CAA Global.
Details of pass standards for CAA exams will be
published in due course.

Pearson | VUE

You can sit the exam at
one of the many centres
worldwide managed by
Pearson VUE.

You can find details of
your local exam centre on
their website by using their
regional contact details:

www.pearsonvue.com/
caaglobal
Specimen exam paper

A specimen exam paper for Module 3, with sample exam questions, is given in Appendix 2. This will show you the types of questions which will be asked in the exam.

Please note

Only 20 sample questions are given in the Specimen exam paper. There will be a total of 60 questions in the Module 3 exam you sit.

The questions found in the specimen paper will not be included in the Module 3 exam.

Resources available at the exam centre

Calculators

There is only one authorised calculator for all the CAA exams:

- **Texas Instruments TI-30 Multiview** (with or without suffix).

You should bring your own calculator with you to the exam. It will be checked by exam centre staff, and the memory will be cleared.

If you bring a different calculator model it must be left in the locker with your other personal belongings.

An on-screen scientific calculator will also be available for you to use during the exam. However, some students have reported that they found this on-screen calculator difficult or cumbersome to use, and so you may prefer to take your own TI-30 calculator to the exam with you.

The TI-30 Multiview calculator is available to buy from shops or online retailers.

To clear the memory and reset the calculator:

| 2nd [reset] 2 or on & clear | Resets the TI-30XS MultiViewTM Calculator. Returns unit to default settings; clears memory variables, pending operations, all entries in history, and statistical data; clears the constant feature, K and Ans |

Please note

Only 20 sample questions are given in the Specimen exam paper. There will be a total of 60 questions in the Module 3 exam you sit.

The questions found in the specimen paper will not be included in the Module 3 exam.
Making notes during the exam

You will be provided with an erasable note board at the centre to use during the exam.

You will only be given one board at a time but are entitled to as many as you need during the exam, and you will be able to keep these at your desk for the duration of the exam. You should ask the supervisor for more if needed. The Pearson VUE staff will not provide you with an eraser for the note boards.

The note boards will be collected by Pearson VUE staff at the end of the exam.

On occasion you may instead be given scrap paper to make notes on.

Formulae and Tables for actuarial examinations

The book of Formulae and Tables for examinations has been published to help students who sit actuarial exams.

The book gives you formulae for:

- selected mathematical and statistical methods,
- calculus, time series and economic models,
  and many other topics.

There are also tables for:

- compound interest calculations,
- selected statistical distributions, and
- other actuarial calculations.

You should make yourself familiar with these tables and formulae during your exam preparation.

You will not be able to use your own copy of the book during your exam, but a PDF copy will be available on your exam screen for you to use.
Appendix 1
Syllabus: Module 3 - Long Term Actuarial Mathematics

Aim

The aim of the Long Term Actuarial Mathematics syllabus is to provide a grounding in the mathematical techniques which can be used to model and value cashflows dependent on death, survival, or other uncertain risks arising in pensions and life insurance.

Note: Where appropriate in the TOPICS within this syllabus only gross premium and gross premium reserves are covered (net premium reserves are not required).

Learning objectives

(i) Define the following terms:
   a) whole life assurance  
   b) term assurance  
   c) pure endowment  
   d) endowment assurance  
   e) critical illness assurance  
   f) whole life level annuity  
   g) temporary level annuity  
   h) guaranteed level annuity  
   i) premium  
   j) benefit

   including assurance and annuity contracts where the benefits are deferred.
(ii) Define the following probabilities: \( n|m q_x \), \( n|q_x \) together with their select equivalents
\( n|m q_{x+r} \), \( n|q_{x+r} \).

(iii) Determine expressions in the form of sums for the expected present value of benefit payments under each contract in 1(i) above, assuming that death benefits are payable at the end of the year of death and that annuities are paid annually in advance or in arrears. Where appropriate, simplify these expressions into a form suitable for evaluation by table look-up or other means.

(iv) Define the symbols \( A_x, A_{x:n} \), \( A_{x:n}^\frac{1}{x} \), \( a_x, a_{x:n} \), \( m|a_{x:n} \), \( \ddot{a}_x, \ddot{a}_{x:n} \), \( m|\ddot{a}_{x:n} \) and their select and continuous equivalents.

(v) Apply the annuity factors in 1.(iv) above to allow for the possibility that payments are more frequent than annual but less frequent than continuous.

(vi) Describe the different types of mortality table and be able to use them including ways in which future improvements can be allowed for.

**Topic 2**

Practical methods of evaluating the expected values of contracts as defined in Topic 1

Indicative study and assessment weighting 15%

Learning objectives

(i) Describe the life table functions \( l_x \) and \( d_x \) and their select equivalents \( l_{x+r} \), and \( d_{x+r} \).

(ii) Define the following life table probabilities in terms of the functions in 2(i):
\( n p_x, n q_x, n|m q_x \) together with their select equivalents \( n|p_{x+r}, n|q_{x+r} \), and \( n|m|q_{x+r} \).

(iii) Define the expected values in objective 1 (iii) in terms of the functions in 2 (i) and 2 (ii).

(iv) Evaluate the expected values in objective 1 (iii) by table look-up.

(v) Determine approximations for, and hence evaluate, the expected values in objective 1 (iv) in terms of those in objective 1 (iii).
Learning objectives

(i) List the types of expenses incurred in writing a life insurance contract.

(ii) Describe the types of expenses incurred in writing a life insurance contract.

(iii) Describe the influence of inflation on the expenses listed in 3(ii).

(iv) Describe the premiums and reserves of simple insurance contracts.

(v) Calculate, using ultimate or select mortality, the premiums and reserves of simple insurance contracts.

(vi) Define premiums for the insurance contract benefits in objective 1 (iii).

(Note: Premiums and annuities may be payable annually or continuously. Benefits may be payable at the end of the year of death, immediately on death, annually, or continuous).

(vii) Calculate premiums for the insurance contract benefits in objective 1(iii) using the equivalence principle.

(Note: Premiums and annuities may be payable annually or continuously. Benefits may be payable at the end of the year of death, immediately on death, annually, or continuous).

Learning objectives

(i) State why an insurance company will set up reserves.

(ii) Describe prospective and retrospective reserves.

(iii) Define and evaluate prospective and retrospective reserves in respect of the contracts in objective 1 (iii), with premiums as in 3 (vi) and (vii).
Learning objectives

**Note:** These objectives extend the techniques identified in TOPIC 2.

(i) Calculate the expected present value of an annuity, premium, or benefit payable on death, which increases or decreases.

(ii) Calculate premiums and reserves for contracts with premiums and/or benefits payable on death, which increases or decreases.

(iii) Calculate the expected present value of an annuity, premium or benefit payable on death, which increases or decreases by a constant monetary amount at various times.

(iv) Calculate premiums and reserves for contracts with premiums and/or benefits which increases or decreases by a constant monetary amount at various times.

(v) Define a with profits contract.

(vi) List the types of bonus that may be given to with profits contracts.

(vii) Calculate premiums and reserves for with profits contracts.

**Topic 6**

**Straightforward functions involving two lives**

Indicative study and assessment weighting 10%

Learning objectives

(i) Extend the techniques in TOPICS 1-5 to deal with cashflows dependent upon the death or survival of either or both of two lives.
Learning objectives

(i) Describe methods which can be used to model cashflows contingent upon competing risks.

(ii) Use a multiple-state Markov Model to value the expected cashflows that are contingent upon multiple transition events, in the case of discrete cashflows only, including simple sickness insurance benefits and premiums.

(iii) Use a multiple decrement model to value the expected cashflows that are contingent on multiple decrement events, such as simple pension scheme benefits (note: commutation functions are not required).

(iv) Describe the construction and use of multiple decrement tables, including the relationships with associated single decrement tables, assuming forces of transition are constant over single years of age.

Learning objectives

(i) Describe the technique of discounted emerging costs, for use in pricing, reserving, and assessing profitability.

(ii) Describe the main features of a unit-linked contract.

(iii) Evaluate expected cashflows for whole life, endowment and term assurances, annuities, and unit-linked contracts.

(iv) Apply a profit test to simple annual premium contracts of the types listed in 8(ii) and determine the profit vector, the profit signature, the net present value, and the profit margin.

(v) Show how the profit test may be used to price a product.

(vi) Show how the profit test may be used to determine non-unit reserves for unit-linked contracts.
Learning objectives

(i) State how the following factors contribute to the variation in mortality and morbidity, specifically:
   (a) occupation
   (b) nutrition
   (c) housing
   (d) climate/geography
   (e) education
   (f) genetics

(ii) Define the following forms of selection:
   (a) temporary initial selection
   (b) class selection
   (c) time selection
   (d) spurious selection
   (e) adverse selection

(iii) Explain how selection can be expected to occur amongst individuals taking out each of the main types of life insurance contracts listed in TOPIC 1, or amongst members of large pension schemes.

(iv) Explain how decrements can have a selective effect.

(v) Explain why it is necessary to have different mortality tables for different classes of lives.

(vi) Explain the theoretical basis of the use of risk classification in life insurance.

(vii) Explain the impact of the availability of genetic information on risk classification life insurance.

(viii) Explain the concept of a single figure index and its advantages and disadvantages for summarising and comparing actual experience.

(ix) Define, and illustrate the use of the terms: crude mortality rate, directly standardised and indirectly standardised mortality rate, standardised mortality ratio, and illustrate their use.

END OF SYLLABUS
Appendix 2
Specimen Examination Paper: Module 3 – Long Term Actuarial Mathematics

Please note:
- This Specimen Examination Paper acts as an example of the types of questions that will appear in the Module 3 Exam Paper.
- Although only 20 sample questions are listed in this specimen paper, there will be a total of 60 questions in the actual Module 3 examination.
- The questions found in the Module 3 Specimen Examination Paper will not be included in the actual Module 3 examination.

1. A product pays out a sum assured if, and only if, the policyholder is alive at the end of a fixed term is called.
   Identify this type of product.
   A. an endowment assurance
   B. a pure endowment
   C. a term assurance
   D. a temporary annuity
   Answer: B

2. An individual aged 65 exact purchases an annuity which is to be payable annually in advance starting from age 75 and continuing for twenty years or until earlier death.
   Identify which of the following symbols represents the value of the annuity described.
   A. $10\ddot{a}_{65:20}$
   B. $10\dot{a}_{75:20}$
   C. $\ddot{a}_{25:20}$
   D. $\ddot{a}_{65:20}$
   Answer: A
3. **Define** what is represented by the symbol $l_{x+r}$

   A. The probability that a life aged $[x]$, subject to select mortality, survives until age $[x]+r$.
   
   B. The probability that a life aged $[x]$, subject to select mortality, dies before age $[x]+r$.
   
   C. The number of lives still alive at age $[x]+r$ out of $l_{[x]}$ lives alive at age $[x]$, subject to select mortality.
   
   D. The number of lives who have died by age $[x]+r$ out of $l_{[x]}$ lives alive at age $[x]$, subject to select mortality.

   Answer: **C**

4. Consider the following information:

   $l_{35} = 9894.4299$  $l_{50} = 9712.0728$
   
   $l_{40} = 9856.2863$  $l_{55} = 9557.8179$
   
   $l_{45} = 9801.3123$  $l_{60} = 9287.2164$

   Calculate $15|10q_{35}$ on the basis of this information.

   A. 0.0094
   
   B. 0.0184
   
   C. 0.0429
   
   D. 0.0520

   Answer: **C**

5. An endowment assurance policy of amount 1 is sold. The benefit is payable at the end of the year of death, or on survival to the end of the term, to a life initially aged 35 exact, with a term of 10 years

   **Basis**
   
   Mortality: AM92 Ultimate
   
   Interest: 6% per annum

   Calculate the expected present value of this policy.

   A. 0.007
   
   B. 0.553
   
   C. 0.560
   
   D. 0.648

   Answer: **C**
6. **Identify** which of the following would be classified as a direct expense on a life insurance contract.

   A. taxation on company profits  
   B. commission on policy sales  
   C. rent on office premises  
   D. dividends paid to shareholders  

   Answer: **B**

7. An insurance company issues a whole life assurance policy to a life aged 40 exact. The sum assured of £100,000 is payable immediately on death. Premiums are payable annually in advance for a term of twenty years or until earlier death.

   **Basis**  
   Mortality: AM92 Select  
   Interest: 6% per annum  
   Expenses:  
     Initial: 50% of the first premium payable  
     Renewal: 5% of the second and subsequent premiums payable  
     Terminal: £5,000 payable at the time of the claim  

   **Calculate** the premium payable for this policy, correct to the nearest £10.

   A. £1,020  
   B. £1,050  
   C. £1,180  
   D. £1,210  

   Answer: **D**

8. A man aged 75 exact wishes to buy a single life non-increasing annuity. He has a lump sum of £250,000 available and has approached an insurance company for a quotation. The annuity will be paid annually in advance until the policyholder’s death.

   **Basis**  
   Mortality: AM92 Select  
   Interest: 4% per annum  
   Expenses:  
     Initial: £5,000  
     Renewal: 5% of the annual benefit, payable from the start of the second policy year  

   **Cont...**
Calculate the annual benefit payable, correct to the nearest £100.

A. £27,000
B. £27,500
C. £28,700
D. £29,300

Answer: A

9. A life insurance company issued a number of endowment assurance policies to lives aged 25 exact. The policies had a term of 35 years with a benefit of $40,000 payable at the end of the year of death, or on survival to the end of the term. The level premium, payable annually in advance, was $600 per policy.

Basis
Mortality: AM92 Ultimate
Interest: 4% per annum
Expenses:
Initial: 100% of the first premium payable
Renewal: 2% of the second and subsequent premiums payable
Terminal: £500 payable at the time of the claim

Calculate, correct to the nearest $100, the gross prospective reserve per policy at the end of the tenth year of one of these contracts.

A. $5,700
B. $6,100
C. $6,700
D. $15,500

Answer: B

10. Define, in the context of a life insurance contract, the expected present value of future outgo minus the expected present value of future income.

A. the retrospective reserve
B. the prospective reserve
C. the sum assured
D. the gross premium

Answer: B
11. A contract is issued where the benefit can be increased, at the discretion of the insurer, following a valuation showing a surplus of assets over liabilities.

**Identify** this type of contract.

A. a surplus contract  
B. a unit-linked contract  
C. a reinsurance contract  
D. a with profits contract

Answer: D

12. A company issues whole life assurance contracts to policyholders aged 40 exact. The sum assured, payable at the end of the year of death, is £100,000 in the first year, increasing each year by £5,000. Premiums are payable annually in advance during the policyholder’s lifetime.

**Basis**
- Mortality: AM92 Ultimate  
- Interest: 4% per annum  
- Expenses: Ignore

**Calculate** the annual premium payable, correct to the nearest £10.

A. £1,150  
B. £1,210  
C. £3,080  
D. £3,140

Answer: C

13. A bonus system is used where the annual bonus is a percentage of the sum assured plus bonuses added in the past, such that the total benefit increases exponentially over the term of the policy.

**Identify** this type of bonus system.

A. a compound bonus system  
B. a simple bonus system  
C. an exponential bonus system  
D. an increasing bonus system

Answer: A
14. A male and a female are both aged 65 exact

Basis
Mortality (male life): PMA92C20
Mortality (female life): PFA92C20

Calculate the probability that exactly one of them will be alive in ten years’ time.

A. 0.012
B. 0.082
C. 0.117
D. 0.199

Answer: D

15. A life company issues an annuity contract to a male and female, both aged 55 exact. The benefit is payable annually in advance under the following conditions:

• While both lives are alive the benefit is $10,000 per annum.
• Should the male life die first, the benefit falls to $5,000 per annum, ceasing on the death of the female life.
• Should the female life die first, the benefit ceases to be paid.
• No benefit is payable once both lives have died.

Basis
Mortality (male life): PMA92C20
Mortality (female life): PFA92C20
Interest: 4% per annum
Expenses: Ignore

Calculate the single premium payable, correct to the nearest $100.

A. $160,200
B. $166,900
C. $171,100
D. $177,900

Answer: C
16. In a pension scheme, the forces of decrement at age 55, that apply over the next year, are as follows:

- Retirement: \( \mu_{55}^{r} = 0.600 \)
- Death: \( \mu_{55}^{d} = 0.095 \)
- Withdrawal: \( \mu_{55}^{w} = 0.250 \)

Calculate the dependent rate of death at age 55, \((aq)^{d}_{55}\), correct to 3 decimal places.

A. 0.061  
B. 0.091  
C. 0.162  
D. 0.388  

Answer: A

17. A three year unit-linked policy issued to a life aged 60 exact has the following profit vector:

- \( £454 \)  
- \( £192 \)  
- \( £88 \)

Basis
- Mortality: AM92 Ultimate  
- Interest: 7.5% per annum

Calculate the net present value of the profits of this policy, correct to the nearest £1.

A. £651  
B. £657  
C. £659  
D. £661  

Answer: B

18. Consider the following three statements relating to the reserves held in respect of a unit-linked contract:

I. benefits payable in excess of the unit fund do not affect the non-unit reserves.  
II. the annual management charges increase the unit reserves.  
III. a unit allocation of 100% does not affect the non-unit reserves.

Identify which of the following responses is true.

Cont…
A. Only statements I and II are correct.
B. Only statements I and III are correct.
C. Statements I, II and III are all correct.
D. Only statement III is correct.

Answer: D

19. The following table shows data for the population of a particular city, City B, in Country A, together with the population of Country A.

<table>
<thead>
<tr>
<th>Age range</th>
<th>Number of lives</th>
<th>Number of deaths</th>
<th>Number of lives</th>
</tr>
</thead>
<tbody>
<tr>
<td>21-40</td>
<td>25,000</td>
<td>129</td>
<td>4,000,000</td>
</tr>
<tr>
<td>41-60</td>
<td>35,000</td>
<td>343</td>
<td>2,500,000</td>
</tr>
<tr>
<td>61-80</td>
<td>30,000</td>
<td>812</td>
<td>1,000,000</td>
</tr>
</tbody>
</table>

Calculate, correct to five decimal places, the Directly Standardised Mortality Rate for the population of City B, using Country A as the standard population.

A. 0.00963
B. 0.00980
C. 0.01401
D. 0.01427

Answer: A

20. An event occurs that gives rise to one population having different mortality to another population. The differences in mortality between the two populations are no longer apparent after a period of five years.

Identify the type of selection observed in this situation.

A. Adverse selection.
B. Class selection.
C. Temporary initial selection.
D. Time selection.

Answer: C
Disclaimers:

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