# Audit trail – Vintage Car Scenario

# The following audit trail should be read alongside the model provided.

# Objective

# A vintage car magazine is planning on publishing an article to help its readers understand how difference maintenance regimes can improve the reliability of their older cars. Research published by the Vehicle Welfare Institute suggests that regularly changing engine oil will cause a 5% per annum reduction in car failure rates, and that regularly changing car tyres will initially cause a 3% per annum reduction in car failure rates, falling gradually to 1% per annum after five years.

# The purpose of the model is to perform the following:

#  calculation of the expectation of life until failure for cars currently exactly 10 years old and 15 years old using failure rates, before any allowance for the improvements due either of the two additional scenarios.

#  calculation of the expectation of life until failure for cars currently exactly 10 years old and 15 years old using failure rates, after allowing for the improvements due to the oil change scenario.

#  calculation of the expectation of life until failure for cars currently exactly 10 years old and 15 years old using failure rates, after allowing for the improvements due to the tyre change scenario.

#  a graphical comparison of the failure rates and expectations of life until failure for the base scenario and the two additional scenarios.

# Assumptions

# The following assumptions were made in the model:

#  Assume that 50 is the limiting age under all scenarios.

#  Assume that the failure rate data provided is correct and contains no errors.

#  Assume that the reduction in failure rates provided by the Vehicle Welfare Institute are valid and do not vary by age.

#  Assume that the reductions in failure rates start immediately, i.e. as soon as the campaigns start.

# “Data” worksheet

# This worksheet contains the raw data and performs some simple checks on it.

# The data was provided by the Vehicle Welfare Institute and comprises car failure rates (qx), at age x exact from ages 10 to 50 inclusive. The cells A4 to B44 in this worksheet hold this information.

# In column C a check is included to ensure failure rates increase with age, as we would expect the probability of failure to be greater for older cars.

# In column F checks are performed to show that the failure rates are greater than 0 and less than 1 for each age (as values are all probabilities of failure so should lie within these bounds), to check that the data is complete and does not include any blank cells and to ensure that the value at age 50 is equal to 1 (as this is assumed to be the limiting age at which all cars fail.

# A chart of the failure rates by age has been constructed. A visual inspection shows the rates increase smoothly over time as expected until age 50 at which the probability of failure is set to 1, reflecting that all cars are expected to fail at this age.

# “Parameters” worksheet

# This sheet sets out all the parameters used within the calculations, including:

# Change in car failure rates for the oil change scenario (5% p.a.)

# Change in car failure rates for the tyre change scenario, initially 3% p.a. reducing to 1% p.a. after 5 years.

# For the tyre change scenario the reduction in failure rates reduce uniformly at the end of each year. The reduction in failure rates in intermediate years are calculated by linearly interpolating between the initial improvement rate and the improvement rate reached after 5 years.

# “Base” worksheet

# This worksheet calculates the life expectancy until failure for a car aged 10 and 15 years old.

# In column C, the probabilities of survival (px) are determined as 1 – qx for at each age from 10 to 50, with the qx being referenced from the “Data” worksheet.

# In column D, the cumulative probabilities of survival from age 10 to age 10 + t (t p10) are determined for ages 10 to 50i.e. t = 1 to 40. (Note: no figure is calculated for age 10 (i.e. 0 p10) since this is not required for the expectation of life calculations.)

# The t p10 values are calculated as the product of px (from column C) from ages x = 10 to 10 + t – 1 (using the Excel function PRODUCT to multiply the px’s).

# A similar approach is used to calculate of the cumulative probabilities of survival from age 15 (t p15) in columns E. To calculate these, the product of px from ages 15 to 15 + t is calculated for ages 16 to 50, i.e. t = 1 to 35 in this case.

# In row 46, the life expectancy until failure for a car currently 10 years old and a car currently 15 years old are calculated, by summing the t p10 in column D for age 10, and by summing the t p15 in column E for age 15.

# “Oil Change” worksheet

# This worksheet recalculates the life expectancy until failure using car failure rates which are reduced by an improvement factor of 5% per annum.

# The failure rate improvement for the Oil Change scenario is linked from the “Parameters” tab into cell C4.

# In column D we calculate 1 – the failure rate improvement for each age in order to determine a multiplicative improvement factor to be applied at each age.

# In column E, the adjusted car failure rates at each age from x = 10 to 50 are determined as the base failure rates, taken from the “Base” worksheet for each age, multiplied by the product of the improvement factors (from column D) from ages x = 10 to 10 + t – 1 (using the Excel function PRODUCT to multiply the improvement factors).

# At age 50, the failure rate is set to 1 (i.e. it is assumed that age 50 is unchanged as the limit of the life of a car).

# In columns F, the related adjusted probabilities of survival for a 10 year old car is determined as (1 – adjusted mortality rate).

# In columns G, the related cumulative probabilities of survival and expectations of life for a car currently 10 years old (row 46) are determined using the adjusted probabilities of survival. The calculation approaches used are the same as those used in worksheet “Base”.

# These calculations are then repeated for a car currently aged 15.

# The failure rate improvement for the Oil Change scenario is linked from the age 10 calculations in column C (no values are entered for ages below 15 as these are not required for the age 15 analysis).

# In column D we calculate 1 – the failure rate improvement for each age in order to determine a multiplicative improvement factor to be applied at each age.

# In column L the adjusted mortality rate is calculated with the improvements starting from age 15 instead of 10, using improvement rates in column J taken from the “Parameters” worksheet.

# The adjusted probabilities of survival from age 15 are calculated in columns M from these adjusted mortality rates. The adjusted car failure rates at each age from x = 15 to 50 are determined as the base failure rates, taken from the “Base” worksheet for each age, multiplied by the product of the improvement factors (from column K) from ages x = 15 to 15 + t – 1 (using the Excel function PRODUCT to multiply the improvement factors).

# In column N, the related cumulative probabilities of survival and expectations of life for a car currently 15 years old (row 46) are determined using the adjusted probabilities of survival. The calculation approaches used are the same as those used in worksheet “Base”.

# A check was carried out to ensure that if we set the improvements to zero the results matched the base scenario.

# “Tyre Change” worksheet

# This worksheet recalculates the life expectancy until failure using car failure rates which are reduced by an improvement factor of 3% per annum initially, reducing linearly to 1% per annum after 5 years.

# The failure rate improvements for the Tyre Change scenario are linked from the “Parameters” tab into cells C4-C9. After 5 years the rate linked into cell C9 is used.

# The remaining calculations to determine the life expectancy until failure for a car currently aged 10 in columns D to G are consistent with those described for the “Oil Change” scenario.

# These calculations are then repeated for a car currently aged 15.

# The failure rate improvements for the Tyre Change scenario are linked from the age 10 calculations in column C (no values are entered for ages below 15 as these are not required for the age 15 analysis).

# As the improvement rates are dependent on time not age, the failure rate improvement expected at age 10 exact for the age 10 calculations is applied at age 15 for the age 15 calculations (cell J9 is linked to cell C4).

# The remaining calculations to determine the life expectancy until failure for a car currently aged 15 in columns K to N are consistent with those described for the “Oil Change” scenario.

# A check was carried out to ensure that if we set the improvements to zero the results matched the base scenario.

# A check was carried out to ensure that if we set the improvements to the oil change parameters the results matched the oil change scenario worksheet.

# “Results” worksheet

# This sheet summarises the results of the analysis in the above worksheets.

# The car failure rates for each year after age 10 are summarised in cells B4:E44, linked to values in each of the underlying scenario tabs.

# These rates are used to produce a line graph which shows how the mortality rates vary for each year after age 10 under each of the 3 scenarios modelled.

# The life expectancy until failure for a car aged 10 and a car aged 15 under each scenario is summarised in a table in cells B51:E53.

# This data is used to produce the bar chart presented alongside these results.

# A check is included to ensure that under each scenario the life expectancy is lower at age 15 than at age 10, as we would expect an older car to have a smaller life expectancy.

# A check is included to ensure that under the base scenario the life expectancy is lower at both age 10 and 15 compared to the tyre change scenario, as we would expect life expectancy to increase as a result of applying improvements to the failure rates.

# A check is included to ensure that under the base scenario the life expectancy is lower at both age 10 and 15 compared to the oil change scenario, as we would expect life expectancy to increase as a result of applying improvements to the failure rates.

# A check is included to ensure that under the tyre change scenario the life expectancy is lower at both age 10 and 15 compared to the oil change scenario, as the improvements applied at each future age under the tyre change scenario are smaller than the oil change scenario so would expect a lower expectation of life.

# A check is included to ensure that under each scenario the life expectancy is lower at most 5 years smaller at age 15 than at age 10, as we would expect a car 5 years older car to no more than a 5 years lower life expectancy.